

# Supersymmetric $SO(10)$ Models Inspired by Deconstruction

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## Abstract

We consider 4-dimensional  $N = 1$  supersymmetric  $SO(10)$  models inspired by deconstruction of 5-dimensional  $N = 1$  supersymmetric orbifold  $SO(10)$  models and high dimensional non-supersymmetric  $SO(10)$  models with Wilson line gauge symmetry breaking. We discuss the  $SO(10) \times SO(10)$  models with bi-fundamental link fields where the gauge symmetry can be broken down to the Pati-Salam,  $SU(5) \times U(1)$ , flipped  $SU(5) \times U(1)'$  or the standard model like gauge symmetry. We also propose an  $SO(10) \times SO(6) \times SO(4)$  model with bi-fundamental link fields where the gauge symmetry is broken down to the Pati-Salam gauge symmetry, and an  $SO(10) \times SO(10)$  model with bi-spinor link fields where the gauge symmetry is broken down to the flipped  $SU(5) \times U(1)'$  gauge symmetry. In these two models, the Pati-Salam and flipped  $SU(5) \times U(1)'$  gauge symmetry can be further broken down to the standard model gauge symmetry, the doublet-triplet splittings can be obtained by the missing partner mechanism, and the proton decay problem can be solved. We also study the gauge coupling unification. We briefly comment on the interesting variation models with gauge groups  $SO(10) \times SO(6)$  and  $SO(10) \times$  flipped  $SU(5) \times U(1)'$  in which the proton decay problem can be solved.

## I. INTRODUCTION

Supersymmetry (SUSY) provides an elegant solution to the gauge hierarchy problem, and grand unified theories (GUTs) give us a simple understanding of the quantum numbers of the standard model (SM) fermions. In addition, the success of gauge coupling unification in the minimal supersymmetric standard model (MSSM) strongly supports the possibility of the SUSY GUT. Other appealing features include that the electroweak symmetry breaking is induced by radiative corrections due to the large top quark Yukawa coupling, and that tiny neutrino masses can be naturally explained by the see-saw mechanism. Therefore, SUSY GUT is one of the most promising candidates that describe the known fundamental interactions in nature except gravity. However, there are problems in the 4-dimensional SUSY GUT: the doublet-triplet splitting problem, the proton decay problem, the fermion mass problem, and the GUT gauge symmetry breaking mechanism.

During the last few years, orbifold GUTs have been studied extensively [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The main point is that the supersymmetric GUT models exist in 5 or 6 dimensional space-time, and they are broken down to 4-dimensional  $N = 1$  supersymmetric SM like models for the zero modes through compactification on various orbifolds. This is because the orbifold parity projects out the zero modes of some components in vector multiplet and hypermultiplets. The GUT gauge symmetry breaking problem, the doublet-triplet splitting problem and the fermion mass problem have been solved elegantly by orbifold projections. The proton decay problem can be solved because we can define a continuous  $U(1)_R$  symmetry. Other interesting phenomenology, like flavour symmetry from the  $R$  symmetry, gauge-fermion unification, gauge-Higgs unification, and gauge-Yukawa unification, have also been studied [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

On the other hand, deconstruction was proposed about three year ago to latticize the gauge theories in higher dimensions [18, 19, 20]. The idea of deconstruction is interesting because it gives a UV completion to the higher dimensional theories. Applying this idea to orbifold SUSY GUTs, we are able to construct interesting 4-dimensional SUSY GUTs where the problems in the usual 4-dimensional models can be solved, in other words, the merits of orbifold GUTs can be preserved [21, 22, 23]. In addition, inspired by deconstruction of orbifold SUSY GUTs, we can construct new models which can not be obtained from orbifold GUTs while still retain the nice properties in orbifold models. Deconstruction of

the orbifold  $SU(5)$  models were discussed in Ref. [21, 22, 23]. In these models the doublet-triplet splitting problem and the proton decay problem can be solved. Deconstruction may also provide insight into fermion masses and mixings [21]. Since the number of fields in these models is finite, the corrections to gauge couplings can be reliably calculated, and there exist threshold corrections to the differential runnings of the gauge couplings [21, 22]. Elements of deconstruction can be found in earlier papers [24].

In this paper, we construct 4-dimensional  $N = 1$  supersymmetric  $SO(10)$  models inspired by deconstruction of the 5-dimensional  $N = 1$  supersymmetric orbifold  $SO(10)$  models and high dimensional non-supersymmetric  $SO(10)$  models with Wilson line gauge symmetry breaking. We study  $SO(10) \times SO(10)$  models with bi-fundamental link fields where the gauge symmetry can be broken down to the Pati-Salam (PS),  $SU(5) \times U(1)$ , flipped  $SU(5) \times U(1)'$  or the SM like gauge symmetry. However, we need to fine-tune the superpotential in the models, in order to have the doublet-triplet splitting. In addition, we propose an  $SO(10) \times SO(6) \times SO(4)$  model where the gauge symmetry can be broken down to PS gauge symmetry, and an  $SO(10) \times SO(10)$  model with bi-spinor link fields  $(\mathbf{16}, \overline{\mathbf{16}})$  and  $(\overline{\mathbf{16}}, \mathbf{16})$ , in which the gauge symmetry is broken down to the flipped  $SU(5) \times U(1)'$  gauge symmetry. In these models, the gauge symmetry can be further broken down to the SM gauge symmetry, and the doublet-triplet splitting is naturally realized through the missing partner mechanism. The proton decay due to dimension-5 operators is thus negligible and the proton lifetime due to dimension-6 operators is well above the current experimental bound because the GUT scale is at least a few times  $10^{16}$  GeV. Therefore, there is no proton decay problem. We also discuss the gauge coupling unification in these two models. Furthermore, we briefly comment on the interesting variation models with gauge groups  $SO(10) \times SO(6)$  and  $SO(10) \times$  flipped  $SU(5) \times U(1)'$  where the proton decay problem can be solved, and the  $SO(10) \times SO(10)$  model with bi-spinor link fields in which the gauge symmetry is broken down to the  $SU(5) \times U(1)$  gauge symmetry.

We first give a brief review of the orbifold SUSY GUTs and the non-SUSY GUTs with Wilson line gauge symmetry breaking, and deconstruction of both types of models in Section II. We then discuss the models where the gauge symmetry can be broken down to the diagonal PS gauge symmetry in Section III. We comment on the models inspired by deconstructions of non-SUSY GUTs with Wilson line gauge symmetry breaking in Section IV. In Section V, we consider gauge symmetry breaking with bi-spinor link fields. We summarize

our results in Section VI.

## II. BRIEF REVIEW OF HIGH DIMENSIONAL GUT BREAKING AND DECONSTRUCTION

### A. 5-Dimensional Orbifold Supersymmetric GUTs

In the 5-dimensional orbifold SUSY GUTs, the 5-dimensional manifold is factorized into the product of ordinary 4-dimensional Minkowski space-time  $M^4$  and the orbifold  $S^1/(Z_2 \times Z'_2)$ . The corresponding coordinates are  $x^\mu$  ( $\mu = 0, 1, 2, 3$ ) and  $y \equiv x^5$ . The radius for the fifth dimension is  $R$ . The orbifold  $S^1/(Z_2 \times Z'_2)$  is obtained by  $S^1$  moduloing the equivalent class

$$P : y \sim -y, \quad P' : y' \sim -y', \quad (1)$$

where  $y' \equiv y - \pi R/2$ . There are two fixed points,  $y = 0$  and  $y = \pi R/2$ .

The  $N = 1$  supersymmetric theory in 5-dimension have 8 real supercharges, corresponding to  $N = 2$  supersymmetry in 4-dimension. In terms of the physical degrees of freedom, the vector multiplet contains a vector boson  $A_M$  with  $M = 0, 1, 2, 3, 5$ , two Weyl gauginos  $\lambda_{1,2}$ , and a real scalar  $\sigma$ . In the 4-dimensional  $N = 1$  supersymmetry language, it contains a vector multiplet  $V \equiv (A_\mu, \lambda_1)$  and a chiral multiplet  $\Sigma \equiv ((\sigma + iA_5)/\sqrt{2}, \lambda_2)$  which transform in the adjoint representation of group  $G$ . The 5-dimensional hypermultiplet consists of two complex scalars  $\phi$  and  $\phi^c$ , and a Dirac fermion  $\Psi$ . It can be decomposed into two chiral multiplets  $\Phi(\phi, \psi \equiv \Psi_R)$  and  $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$ , which are in the conjugate representations of each other under the gauge group.

The general action for the group  $G$  gauge fields and their couplings to the bulk hypermultiplet  $\Phi$  is [25]

$$\begin{aligned} S = & \int d^5x \frac{1}{kg^2} \text{Tr} \left[ \frac{1}{4} \int d^2\theta (W^\alpha W_\alpha + \text{H.C.}) \right. \\ & + \int d^4\theta \left( (\sqrt{2}\partial_5 + \bar{\Sigma})e^{-V}(-\sqrt{2}\partial_5 + \Sigma)e^V + \partial_5 e^{-V} \partial_5 e^V \right) \\ & + \int d^5x \left[ \int d^4\theta (\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi) \right. \\ & \left. \left. + \int d^2\theta \left( \Phi^c (\partial_5 - \frac{1}{\sqrt{2}}\Sigma) \Phi + \text{H.C.} \right) \right] \right]. \quad (2) \end{aligned}$$

Under the parity operator  $P$ , the vector multiplet transforms as

$$V(x^\mu, y) \rightarrow V(x^\mu, -y) = PV(x^\mu, y)P^{-1} , \quad (3)$$

$$\Sigma(x^\mu, y) \rightarrow \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1} . \quad (4)$$

For the hypermultiplet  $\Phi$  and  $\Phi^c$ , we have

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P^{l_\Phi} \Phi(x^\mu, y) (P^{-1})^{m_\Phi} , \quad (5)$$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P^{l_\Phi} \Phi^c(x^\mu, y)^{m_\Phi} , \quad (6)$$

where  $\eta_\Phi$  is  $\pm$ ,  $l_\Phi$  and  $m_\Phi$  are respectively the numbers of the fundamental index and anti-fundamental index for the bulk multiplet  $\Phi$  under the bulk gauge group  $G$ . For example, if  $G$  is an  $SU(N)$  group, for fundamental representation,  $l_\Phi = 1$ ,  $m_\Phi = 0$ , and for adjoint representation,  $l_\Phi = 1$ ,  $m_\Phi = 1$ . Moreover, the transformation properties for the vector multiplet and hypermultiplets under  $P'$  are the same as those under  $P$ .

For  $G = SU(5)$ , to break the  $SU(5)$  gauge symmetry, we choose the following  $5 \times 5$  matrix representations for the parity operators  $P$  and  $P'$

$$P = \text{diag}(+1, +1, +1, +1, +1) , \quad P' = \text{diag}(+1, +1, +1, -1, -1) . \quad (7)$$

Under the  $P'$  parity, the gauge generators  $T^\alpha$  ( $\alpha = 1, 2, \dots, 24$ ) for  $SU(5)$  are separated into two sets:  $T^a$  are the generators for the SM gauge group, and  $T^{\hat{a}}$  are the generators for the broken gauge group

$$P T^a P^{-1} = T^a , \quad P T^{\hat{a}} P^{-1} = T^{\hat{a}} , \quad (8)$$

$$P' T^a P'^{-1} = T^a , \quad P' T^{\hat{a}} P'^{-1} = -T^{\hat{a}} . \quad (9)$$

The zero modes of the  $SU(5)/SM$  gauge bosons are projected out, thus, the 5-dimensional  $N = 1$  supersymmetric  $SU(5)$  gauge symmetry is broken down to the 4-dimensional  $N = 1$  supersymmetric SM gauge symmetry for the zero modes. For the zero modes and KK modes, the 4-dimensional  $N = 1$  supersymmetry is preserved on the 3-branes at the fixed points, and only the SM gauge symmetry is preserved on the 3-brane at  $y = \pi R/2$ .

For  $G = SO(10)$ , the generators  $T^\alpha$  of  $SO(10)$  are imaginary antisymmetric  $10 \times 10$  matrices. In terms of the  $2 \times 2$  identity matrix  $\sigma_0$  and the Pauli matrices  $\sigma_i$ , they can be

written as tensor products of  $2 \times 2$  and  $5 \times 5$  matrices,  $(\sigma_0, \sigma_1, \sigma_3) \otimes A_5$  and  $\sigma_2 \otimes S_5$  as a complete set, where  $A_5$  and  $S_5$  are the  $5 \times 5$  real anti-symmetric and symmetric matrices. The generators of the  $SU(5) \times U(1)$  are

$$\begin{aligned} \sigma_0 \otimes A_3, \quad \sigma_0 \otimes A_2, \quad \sigma_0 \otimes A_X \\ \sigma_2 \otimes S_3, \quad \sigma_2 \otimes S_2, \quad \sigma_2 \otimes S_X, \end{aligned} \tag{10}$$

the generators for flipped  $SU(5) \times U(1)'$  are

$$\begin{aligned} \sigma_0 \otimes A_3, \quad \sigma_0 \otimes A_2, \quad \sigma_1 \otimes A_X \\ \sigma_2 \otimes S_3, \quad \sigma_2 \otimes S_2, \quad \sigma_3 \otimes A_X, \end{aligned} \tag{11}$$

and the generators for  $SU(4)_C \times SU(2)_L \times SU(2)_R$  are

$$\begin{aligned} (\sigma_0, \sigma_1, \sigma_3) \otimes A_3, \quad (\sigma_0, \sigma_1, \sigma_3) \otimes A_2, \\ \sigma_2 \otimes S_3, \quad \sigma_2 \otimes S_2, \end{aligned} \tag{12}$$

where  $A_3$  and  $S_3$  are respectively the diagonal blocks of  $A_5$  and  $S_5$  that have indices 1, 2, and 3, while the diagonal blocks  $A_2$  and  $S_2$  have indices 4 and 5.  $A_X$  and  $S_X$  are the off diagonal blocks of  $A_5$  and  $S_5$ .

We choose the  $10 \times 10$  matrix for  $P$  as

$$P = \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1). \tag{13}$$

To break the  $SO(10)$  down to  $SU(5) \times U(1)$ , we choose

$$P' = \sigma_2 \otimes \text{diag}(1, 1, 1, 1, 1), \tag{14}$$

to break the  $SO(10)$  down to flipped  $SU(5) \times U(1)'$ , we choose

$$P' = \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1), \tag{15}$$

and to break the  $SO(10)$  down to the PS gauge symmetry, we choose

$$P' = \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1). \tag{16}$$

For the zero modes, the 5-dimensional  $N = 1$  supersymmetric  $SO(10)$  gauge symmetry is broken down to the 4-dimensional  $N = 1$  supersymmetric  $SU(5) \times U(1)$ , flipped  $SU(5) \times U(1)'$  and the PS gauge symmetries. Including the KK modes, the 3-branes at the fixed

points preserve the 4-dimensional  $N = 1$  supersymmetry, and the gauge symmetry on the 3-brane at  $y = \pi R/2$  is  $SU(5) \times U(1)$ , flipped  $SU(5) \times U(1)'$  and the PS gauge symmetries, for different choices of  $P'$ .

We can also break the  $SO(10)$  down to the SM like  $(SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X)$  gauge symmetry, by choosing the following matrix representations for  $P$  and  $P'$

$$P = \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1), \quad P' = \sigma_2 \otimes \text{diag}(1, 1, 1, 1, 1), \quad (17)$$

or

$$P = \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1), \quad P' = \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1), \quad (18)$$

or

$$P = \sigma_2 \otimes \text{diag}(1, 1, 1, 1, 1), \quad P' = \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1). \quad (19)$$

## B. High Dimensional non-Supersymmetric GUTs with Wilson Line Gauge Symmetry Breaking

Other than the orbifold GUTs, Wilson line gauge symmetry breaking can be applied to break high dimensional gauge symmetries [26]. Because supersymmetry can not be broken with this mechanism, only non-supersymmetric models are considered.

First, let us consider the 5-dimensional space-time  $M^4 \times S^1$  with coordinates  $x^\mu$ , and  $y \equiv x^5$ . The radius for the fifth dimension is  $R$ . The gauge fields are denoted as  $A_M(x^\mu, y)$  where  $M = 0, 1, 2, 3, 5$ . Because  $Z_2$  belongs to the fundamental group of  $S^1$ , we can define a  $Z_2$  parity operator  $P_y$  for a generic bulk multiplet  $\Phi(x^\mu, y)$

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, y + 2\pi R) = \eta_\Phi P_y^{l_\Phi} \Phi(x^\mu, y) (P_y^{-1})^{m_\Phi}, \quad (20)$$

where  $\eta_\Phi = \pm 1$  and  $P_y^2 = 1$ .

For a  $SU(5)$  model, if we choose  $P_y$  to be equal to  $P'$  in Eq. (7), the  $SU(5)$  gauge symmetry is broken down to the SM gauge symmetry for the zero modes.

For a  $SO(10)$  model, if we choose  $P_y$  the same as  $P'$  in Eqs. (14), (15) or (16), the  $SO(10)$  gauge symmetry is broken down to the  $SU(5) \times U(1)$ , flipped  $SU(5) \times U(1)'$ , or the PS gauge symmetry respectively, for the zero modes.

Similarly, we can consider the 6-dimensional space-time  $M^4 \times S^1 \times S^1$ , with coordinates  $x^\mu$ ,  $y \equiv x^5$ , and  $z \equiv x^6$ . The radii for the fifth and sixth dimensions are  $R_1$  and  $R_2$  respectively. We can define two  $Z_2$  parity operators  $P_y$  and  $P_z$  for a generic bulk multiplet  $\Phi(x^\mu, y, z)$

$$\Phi(x^\mu, y, z) \rightarrow \Phi(x^\mu, y + 2\pi R_1, z) = \eta_\Phi^y P_y^{l_\Phi} \Phi(x^\mu, y, z) (P_y^{-1})^{m_\Phi}, \quad (21)$$

$$\Phi(x^\mu, y, z) \rightarrow \Phi(x^\mu, y, z + 2\pi R_2) = \eta_\Phi^z P_z^{l_\Phi} \Phi(x^\mu, y, z) (P_z^{-1})^{m_\Phi}, \quad (22)$$

where  $\eta_\Phi^y = \pm 1$ ,  $\eta_\Phi^z = \pm 1$ ,  $P_y^2 = 1$ , and  $P_z^2 = 1$ .

By choosing the following  $P_y$  and  $P_z$  exactly the same as  $P$  and  $P'$  in Eqs. (17), (18) or (19), we can break the  $SO(10)$  down to the SM like gauge symmetry.

Inspired by the high dimensional non-supersymmetric GUTs with Wilson line gauge symmetry breaking, we can construct the 4-dimensional  $N = 1$  supersymmetric  $G^N$  models where the gauge symmetry  $G^N$  can be broken down to a diagonal subgroup of  $G$  or the SM like gauge symmetry. SUSY GUT models with Wilson line gauge symmetry breaking have the problem that the 5-dimensional  $N = 1$  supersymmetry or the 6-dimensional  $N = 2$  supersymmetry can not be broken down to the 4-dimensional  $N = 1$  supersymmetry. However, we can start from the 4-dimensional  $N = 1$  supersymmetry, which is exactly the advantage of the 4-dimensional supersymmetric GUT models inspired by the deconstructions.

### C. Deconstruction of Orbifold GUTs and Wilson Line Gauge symmetry Breaking

For the deconstruction of the orbifold SUSY GUTs, we break the  $G^{N-1} \times G_s$  to a diagonal  $G_s \subset G$  as the following. Suppose there are  $N$  nodes, and the gauge symmetry on the first  $(N-1)$  nodes is  $G$  while the gauge symmetry on the last node is  $G_s$ . The nodes are connected with  $(N-1)$  bi-fundamental fields  $U_i$ , and there is no link field between the first and last nodes. When the  $U_i$ 's acquire uniform VEVs,  $\langle U_i \rangle = (v/\sqrt{2}) \text{diag}(1, 1, \dots, 1)$ , for all  $i$ 's. The gauge bosons of the group  $G_s$  have a  $N \times N$  mass matrix, which has a zero eigenvalue, while the  $G/G_s$  gauge bosons have a  $(N-1) \times (N-1)$  mass matrix, which does not have a zero eigenvalue [21]. As the  $G/G_s$  gauge bosons become heavy,  $G^{N-1} \times G_s$  is effectively broken down to  $G_s$  for the massless fields. Additional fields will be needed on the first and  $N$ -th nodes to cancel anomalies. Alternatively,  $N$  bi-fundamental fields  $U_i$  connect the  $N$



nodes to form a loop. With the same uniform VEVs, the  $G_s$  gauge bosons have  $N \times N$  mass matrices, while the  $G/G_s$  gauge bosons have  $(N - 1) \times (N - 1)$  mass matrices. Only one set of  $G_s$  gauge bosons remain massless. The anomalies are always canceled in this setup, thus no additional field is required.

From now on we always discuss the simplified case of two copies of the initial  $G$  group  $G_1 \times G_2$  where  $G_2 \subseteq G_1$ , with  $A_\mu^\alpha$  and  $T_\alpha$  being the gauge fields and generators of  $G_1$ , and  $B_\mu^\beta$  and  $T_\beta$  the gauge fields and generators of  $G_2$ . We assume that the gauge couplings are equal at the breaking scale for simplicity. The generalization to more copies of the  $G$  group is straightforward. With  $U_1$  and  $U_2$  being two bi-fundamental fields, the covariant derivatives are

$$\begin{aligned} D_\mu U_1 &= \partial_\mu U_1 - iA_\mu^\alpha T_\alpha U_1 + iB_\mu^\beta T_\beta U_1, \\ D_\mu U_2 &= \partial_\mu U_2 + iA_\mu^\alpha T_\alpha U_2 - iB_\mu^\beta T_\beta U_2, \end{aligned} \quad (23)$$

and the effective action for the scalar fields is

$$\begin{aligned} S = \int d^4x \left\{ -\frac{1}{4g^2} \text{Tr} F_1^2 + \text{Tr}[(D_\mu U_1)^\dagger D^\mu U_1] - \frac{1}{4g^2} \text{Tr} F_2^2 + \right. \\ \left. \text{Tr}[(D_\mu U_2)^\dagger D^\mu U_2] + \dots \right\}. \end{aligned} \quad (24)$$

For simplicity, we only write down the effective action for the scalar fields.

Inspired by the high dimensional non-supersymmetric GUTs with Wilson line gauge symmetry breaking, we can use the VEVs of the bi-fundamental fields that are not commuting with a subset of the generators to break the original group [23]. Let us suppose the initial gauge group is  $G \times G$  and  $G_s$  is a subgroup of  $G$ . The term  $\text{Tr}[(D_\mu U_1)^\dagger D^\mu U_1] + \text{Tr}[(D_\mu U_2)^\dagger D^\mu U_2]$  in Eq. (24) produces a mass matrix of the form

$$\begin{pmatrix} A_\mu^\alpha & B_\mu^\beta \end{pmatrix} \begin{pmatrix} \text{Tr}[U_1^\dagger T_\alpha^\dagger T_{\alpha'} U_1 + T_\alpha^\dagger U_2^\dagger U_2 T_{\alpha'}] & -\text{Tr}[U_1^\dagger T_\alpha^\dagger U_1 T_{\beta'} + T_\alpha^\dagger U_2^\dagger T_{\beta'} U_2] \\ -\text{Tr}[T_\beta^\dagger U_1^\dagger T_{\alpha'} U_1 + U_2^\dagger T_\beta^\dagger U_2 T_{\alpha'}] & \text{Tr}[T_\beta^\dagger U_1^\dagger U_1 T_{\beta'} + U_2^\dagger T_\beta^\dagger T_{\beta'} U_2] \end{pmatrix} \begin{pmatrix} A_\mu^{\alpha'} \\ B_\mu^{\beta'} \end{pmatrix}. \quad (25)$$

After  $U_1$  and  $U_2$  acquire VEVs, the  $U_i$ 's in the mass matrix will be replaced by  $\langle U_i \rangle$ . The VEV  $\langle U_i \rangle$  has the property that  $\langle U_i \rangle^\dagger \langle U_i \rangle$  is always proportional to the identity matrix. The normalized generators satisfy the relation  $\text{Tr}(T_\alpha T_{\alpha'}) = (1/2) \delta_{\alpha\alpha'}$  for  $SU(N)$  or  $\text{Tr}(T_\alpha T_{\alpha'}) = 2\delta_{\alpha\alpha'}$  for  $SO(N)$ , thus the diagonal terms in matrix in Eq. (25) are always proportional to  $\delta_{\alpha\alpha'}$ , while the off-diagonal terms depend on the commutation

relations between the VEVs of the bi-fundamental fields and the generators. If the VEV of  $U_1$  commutes with all the generators of  $G$ , while the VEV of the  $U_2$  commutes with the generators of  $G_s$  group,  $T_a$ , and anti-commutes with those of  $G/G_s$ ,  $T_{\hat{a}}$

$$\langle U_1 \rangle^\dagger T_a \langle U_1 \rangle = T_a \quad \text{and} \quad \langle U_1 \rangle^\dagger T_{\hat{a}} \langle U_1 \rangle = T_{\hat{a}}, \quad (26)$$

$$\langle U_2 \rangle^\dagger T_a \langle U_2 \rangle = T_a \quad \text{and} \quad \langle U_2 \rangle^\dagger T_{\hat{a}} \langle U_2 \rangle = -T_{\hat{a}}, \quad (27)$$

the mass matrices become proportional to

$$\begin{pmatrix} A_\mu^a & B_\mu^b \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} A_\mu^a \\ B_\mu^b \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A_\mu^{\hat{a}} & B_\mu^{\hat{b}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A_\mu^{\hat{a}} \\ B_\mu^{\hat{b}} \end{pmatrix}, \quad (28)$$

respectively. While the former matrix has a 0 eigenvalue, the latter does not. Thus the gauge symmetry  $G \times G$  is broken down to the diagonal subgroup  $G_s$ .

Before presenting our models, we would like to emphasize that we consider the 4-dimensional  $N = 1$  supersymmetry in all the models discussed in this paper.

### III. BREAKING VIA PATI-SALAM

One way to realize the deconstruction of orbifold supersymmetric  $SO(10)$  models is to start with the initial gauge group  $SO(10) \times SO(6) \times SO(4)$ . It is broken down to the diagonal PS gauge symmetry by bifundamental link fields. The doublet-triplet splitting can be achieved by the missing partner mechanism, and the proton decay problem is solved because the gauge coupling unification scale is a few times  $10^{16}$  GeV. A simple variation of this model is the  $SO(10) \times SO(6)$  model. In addition, we can consider the  $SO(10) \times SO(10)$  model, where the gauge symmetry can be broken down to the diagonal PS gauge symmetry, which is inspired by the deconstruction of 5-dimensional Wilson line gauge symmetry breaking. However, the doublet-triplet splitting problem can not be solved without fine-tuning in superpotential.

#### A. $SO(10) \times SO(6) \times SO(4)$ Model

To break  $SO(10) \times SO(6) \times SO(4)$  to the diagonal PS gauge symmetry, we rely on two isomorphisms,  $SO(6) \cong SU(4)$  and  $SO(4) \cong SU(2) \times SU(2)$ . We introduce the bi-

fundamental fields  $U_1$  and  $U_2$  with the following quantum numbers

|       | $SO(10)$  | $SO(6) \times SO(4)$ |
|-------|-----------|----------------------|
| $U_1$ | <b>10</b> | <b>(6, 1)</b>        |
| $U_2$ | <b>10</b> | <b>(1, 4)</b>        |

(29)

Suppose, for example, we have the superpotential

$$W = S_1 (U_1 U_1 - 3v^2) + S_2 (U_2 U_2 - 2v^2) , \quad (30)$$

where  $S_1$  and  $S_2$  are singlets, and the link fields acquire VEVs

$$\langle U_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} I_{6 \times 6} \\ 0_{4 \times 6} \end{pmatrix} , \quad \langle U_2 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0_{6 \times 4} \\ I_{4 \times 4} \end{pmatrix} , \quad (31)$$

where  $I_{i \times i}$  is the  $i \times i$  identity matrix, and  $0_{i \times j}$  is a  $i \times j$  matrix where all the entries are zero. The  $2 \times 2$  mass matrix for the PS gauge bosons is

$$m_{PS}^2 = 2g^2 v^2 X_1 , \quad X_1 \equiv \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} , \quad (32)$$

with eigenvalues  $0, 4g^2 v^2$ . The masses of the  $SO(10)/(SO(6) \times SO(4))$  gauge bosons are simply  $2g^2 v^2$ . If generalized to a model with gauge group  $SO(10)^{N-1} \times SO(6) \times SO(4)$ , the PS gauge bosons have a  $N \times N$  mass matrix which has a zero eigenvalue, whereas the non-PS gauge bosons have a  $(N-1) \times (N-1)$  mass matrix, for which the zero eigenvalue is absent.

Three families of the SM fermions form three **16** spinor representations of  $SO(10)$ , and we introduce a 10-dimensional Higgs field  $H_{10}$ . To break the PS gauge symmetry and at the same time require a left-right symmetry, *i.e.*, the coupling  $\alpha_L$  of the  $SU(2)_L$  equals to  $\alpha_R$  of the  $SU(2)_R$  above the PS gauge symmetry unification scale ( $M_{PS}$ ), we introduce the fields  $\Sigma_h, \bar{\Sigma}_h, \Sigma_f, \bar{\Sigma}_f, C_1, C_2, S$  and  $S'$ . Their quantum numbers under the gauge symmetry  $SO(10) \times SU(4) \times SU(2)_L \times SU(2)_R$  are given in the following

|                  | $SO(10)$ | $SU(4) \times SU(2)_L \times SU(2)_R$ |
|------------------|----------|---------------------------------------|
| $\Sigma_h$       | <b>1</b> | <b>(4, 1, 2)</b>                      |
| $\bar{\Sigma}_h$ | <b>1</b> | <b>(<math>\bar{4}</math>, 1, 2)</b>   |
| $\Sigma_f$       | <b>1</b> | <b>(4, 2, 1)</b>                      |
| $\bar{\Sigma}_f$ | <b>1</b> | <b>(<math>\bar{4}</math>, 2, 1)</b>   |
| $C_1, C_2$       | <b>1</b> | <b>(6, 1, 1)</b>                      |
| $S, S'$          | <b>1</b> | <b>(1, 1, 1)</b>                      |

(33)

Note that  $\Sigma_h, \bar{\Sigma}_h, \Sigma_f, \bar{\Sigma}_f$  can form one pair of **16** and  **$\bar{16}$**  under  $SO(10)$ . The PS gauge symmetry is broken down to the SM gauge symmetry when the right-handed neutrino scalar component in  $\Sigma_h$  and its charge conjugation in  $\bar{\Sigma}_h$  obtain VEVs at the  $M_{PS}$  scale.

The superpotential is

$$W = y_1 S(\Sigma_h \bar{\Sigma}_h - M_{PS}^2) + M_{PS} \Sigma_f \bar{\Sigma}_f + y_2 \Sigma_h C_2 \Sigma_h + y_3 \bar{\Sigma}_h C_2 \bar{\Sigma}_h + y_4 H_{10} U_1 C_1 + y_5 S' H_{10} H_{10} + y_{ij} 16_i H_{10} 16_j, \quad (34)$$

where  $y_i$ 's are Yukawa couplings, and  $y_{ij}$ 's are the usual Yukawa couplings for the SM fermions. Moreover, similarly to the doublet-triplet splitting via the missing partner mechanism, the color triplets in  $H_{10}$  and  $C_1$  obtain the GUT scale masses while the doublets are massless because of the superpotential term  $y_4 H_{10} U_1 C_1$ . Subsequently, the proton decay due to the dimension-5 operator is negligible because the mixing of the color triplets through the  $\mu$  term is very small. Limits on the contributions from the dimension-6 proton decay operators require that the gauge coupling unification scale be larger than  $5 \times 10^{15}$  GeV [27], which is obviously satisfied in our model where the GUT scale is at least a few times  $10^{16}$  GeV. Furthermore, the  $\mu$  term for the Higgs doublets can be generated from the superpotential  $y_5 S' H_{10} H_{10}$ , as is similar to the next to the minimal supersymmetric standard model [28]. We also assume that the masses for  $C_2, \Sigma_h$  and  $\bar{\Sigma}_h$  are around  $M_{PS}$ , and the masses for  $C_1$  and Higgs triplets in  $H_{10}$  are about  $\sqrt{2}gv$ . We do not discuss the fermion masses and mixings, as they are out of the scope of this paper.

For simplicity, we assume the universal masses  $M_{SUSY} = 500$  GeV for the supersymmetric particles [34]. From  $M_Z$  to  $M_{SUSY}$ , the gauge couplings evolve the same way as those in the SM, and the beta functions are  $b^0 \equiv (b_1, b_2, b_3) = (41/10, -19/6, -7)$ , where  $b_1, b_2$  and  $b_3$  are for the gauge symmetries  $U(1)_Y, SU(2)_L$  and  $SU(3)_C$ , respectively. From  $M_{SUSY}$  to the PS gauge symmetry unification scale  $M_{PS}$ , the beta functions are just those of the MSSM, *i.e.*,  $b^I = (33/5, 1, -3)$ .

At the PS gauge symmetry unification scale  $M_{PS}$ , the gauge couplings for  $U(1)_Y$  and  $SU(3)_C$  are

$$\alpha_1^{-1}(M_{PS}) = \alpha_1^{-1}(M_Z) - \frac{b_1^0}{2\pi} \log\left(\frac{M_{SUSY}}{M_Z}\right) - \frac{b_1^I}{2\pi} \log\left(\frac{M_{PS}}{M_{SUSY}}\right), \quad (35)$$

$$\alpha_3^{-1}(M_{PS}) = \alpha_3^{-1}(M_Z) - \frac{b_3^0}{2\pi} \log\left(\frac{M_{SUSY}}{M_Z}\right) - \frac{b_3^I}{2\pi} \log\left(\frac{M_{PS}}{M_{SUSY}}\right). \quad (36)$$

The  $SU(2)_R$  gauge coupling  $\alpha_R$  is related to  $\alpha_1$  and  $\alpha_3$  by

$$\alpha_R^{-1}(M_{PS}) = \frac{5}{3}\alpha_1^{-1}(M_{PS}) - \frac{2}{3}\alpha_3^{-1}(M_{PS}) . \quad (37)$$

Above the PS scale, including the contributions from fermions, PS gauge multiplets, two Higgs doublets  $H^D$ ,  $\Sigma_h$ ,  $\bar{\Sigma}_h$ ,  $\Sigma_f$ ,  $\bar{\Sigma}_f$ , and  $C_2$ , the beta functions  $b^{II} \equiv (b_R, b_L, b_4)$  become

$$\begin{aligned} b^{II} &= b(M) + b^{PS}(V) + b(H^D) + b(\Sigma) + b(C_2) \\ &= (6, 6, 6) + (-6, -6, -12) + (1, 1, 0) + (4, 4, 4) + (0, 0, 1) = (5, 5, -1) , \end{aligned} \quad (38)$$

where  $b_R$ ,  $b_L$  and  $b_4$  are the beta functions for  $SU(2)_R$ ,  $SU(2)_L$  and  $SU(4)_C$ , respectively. In the region above  $\sqrt{2}gv$ , one set of  $SO(10)/PS$  gauge multiplets,  $C_1$  and Higgs triplets  $H^T$  in  $H_{10}$  appear, then the beta functions are

$$\begin{aligned} b^{III} &= b^{II} + b^{SO(10)/PS}(V) + b(C_1) + b(H^T) \\ &= (5, 5, -1) + (-18, -18, -12) + (0, 0, 1) + (0, 0, 1) = (-13, -13, -11). \end{aligned} \quad (39)$$

We show the runnings of the gauge couplings near the unification scale in the left panel of Fig. 1. Note that the runnings of  $\alpha_L$  and  $\alpha_R$  coincide above  $M_{PS}$  because we have required the left-right symmetry. The gauge coupling  $\alpha_4$  of  $SU(4)$  unify with  $\alpha_R$  and  $\alpha_L$  at the scale  $M_* = 3.3 \times 10^{16}$  GeV and the PS scale occurs at  $M_{PS} = 1.3 \times 10^{16}$  GeV. Thus, there is no proton decay problem. In the above construction, we maintain the left-right symmetry above the  $M_{PS}$  scale. If we no longer require the left-right symmetry, we can drop the additional fields  $\Sigma_f$  and  $\bar{\Sigma}_f$  from the table in Eq. (33). In this setup, the discussion of the doublet-triplet splitting is unchanged from that of the above symmetric case. The RGE runnings of the gauge couplings are different due to the simpler field content. More specifically, the beta functions are,  $b^{II} = (5, 1, -3)$  and  $b^{III} = (-13, -17, -13)$ . Above  $M_{PS} = 6.9 \times 10^{15}$  GeV,  $\alpha_R$  and  $\alpha_L$  have different RGE runnings, as is evident in Fig. 1 right panel. With a GUT scale of  $M_* = 5.1 \times 10^{16}$  GeV, the proton lifetime is safely above the current experimental bound.

## B. $SO(10) \times SO(6)$ Model

If we want to solve the proton decay problem, we can also consider the model with gauge group  $SO(10) \times SO(6)$ , which can not be obtained but is inspired by the orbifold GUT.

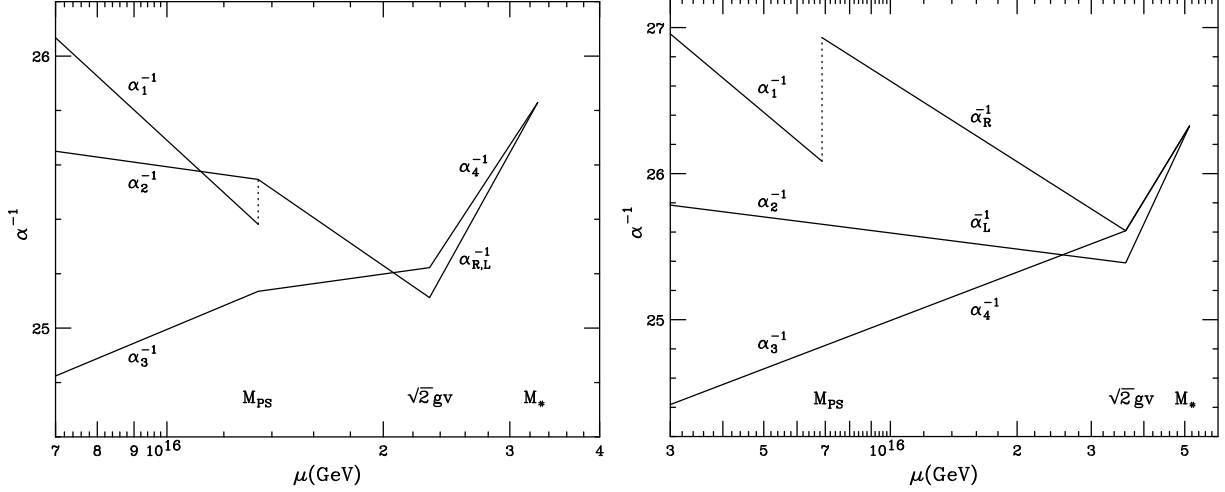


FIG. 1: Left: The gauge coupling unification for the left-right symmetric PS model near  $M_* = 3.3 \times 10^{16}$  GeV. Right: The gauge coupling unification for the left-right non-symmetric PS model near  $M_* = 5.1 \times 10^{16}$  GeV.

To break  $SO(10) \times SO(6)$  to PS gauge symmetry, we introduce bi-fundamental field  $U_1$  as

$$\begin{array}{c|cc} & SO(10) & SO(6) \\ \hline U_1 & \mathbf{10} & (\mathbf{6}, \mathbf{1}) \end{array} \quad (40)$$

The link fields acquire the following VEVs

$$\langle U_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} I_{6 \times 6} \\ 0_{4 \times 6} \end{pmatrix}. \quad (41)$$

The  $SO(10) \times SO(6)$  gauge symmetry is broken down to the PS gauge symmetry.

The discussions on the proton decay, PS gauge symmetry breaking and the gauge coupling unification are similar to those in the above subsection, so, we will not repeat them here. However, we would like to emphasize that the initial gauge group is smaller than that in the above subsection.

### C. $SO(10) \times SO(10)$ Model

Inspired by the deconstruction of 5-dimensional  $SO(10)$  model with Wilson line gauge symmetry breaking, we consider the model with the simplest gauge group  $SO(10) \times SO(10)$ .

Let us introduce bi-vector fields  $U_1$  and  $U_2$  with the following quantum numbers

|       | $SO(10)$  | $SO(10)$  |
|-------|-----------|-----------|
| $U_1$ | <b>10</b> | <b>10</b> |
| $U_2$ | <b>10</b> | <b>10</b> |

(42)

The effective action is the same as that in Eq. (24). If the  $U_1$  and  $U_2$  fields acquire VEVs

$$\langle U_1 \rangle = \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1), \quad \langle U_2 \rangle = \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(-1, -1, -1, 1, 1), \quad (43)$$

the  $2 \times 2$  mass matrix for the PS gauge bosons is

$$m_{PS}^2 = 4g^2 v^2 X_1, \quad (44)$$

with  $X_1$  defined in Eq. (32). The squared masses for the PS group gauge bosons are either 0 or  $8g^2 v^2$ . The mass matrix of the non-PS ( $SO(10)/PS$ ) gauge bosons is

$$m_{NPS}^2 = 4g^2 v^2 X_2, \quad X_2 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (45)$$

It has degenerate eigenstates with squared masses equal to  $4g^2 v^2$ .

Three families of the SM fermions form three **16** spinor representations under the first  $SO(10)$  while they are singlets under the second  $SO(10)$ . We also introduce a Higgs field  $H_{10}$ . To give masses to the color triplets in the Higgs field  $H_{10}$ , we introduce a 10-dimensional field  $H'_{10}$ , and to break the PS gauge symmetry, we introduce the fields  $\Sigma_H$ ,  $\bar{\Sigma}_H$ , and two singlets  $S$  and  $S'$ . The quantum numbers for these particles are

|                  | $SO(10)$                     | $SO(10)$  |
|------------------|------------------------------|-----------|
| $\mathbf{16}_i$  | <b>16</b>                    | <b>1</b>  |
| $H_{10}$         | <b>10</b>                    | <b>1</b>  |
| $H'_{10}$        | <b>1</b>                     | <b>10</b> |
| $\Sigma_H$       | <b>16</b>                    | <b>1</b>  |
| $\bar{\Sigma}_H$ | <b><math>\bar{16}</math></b> | <b>1</b>  |
| $S, S'$          | <b>1</b>                     | <b>1</b>  |

(46)

The superpotential is

$$\begin{aligned} W = & y_1 S (\Sigma_H \bar{\Sigma}_H - M_{PS}^2) + y_2 H_{10} (U_1 - U_2) H'_{10} \\ & + M_* H'_{10} H'_{10} + y_3 S' H_{10} H_{10} + y_{ij} 16_i H_{10} 16_j, \end{aligned} \quad (47)$$

where  $M_*$  is the  $SO(10)$  unification scale. The doublet-triplet splitting can be obtained through the fine-tuning superpotential  $y_2 H_{\mathbf{10}}(U_1 - U_2) H'_{\mathbf{10}}$ , as in the usual  $SU(5)$  model. Because it is less interesting than the usual  $SO(10)$  model from the phenomenological point of view, we do not study it in detail.

#### IV. BREAKING VIA $SU(5) \times U(1)$ OR FLIPPED $SU(5) \times U(1)'$

For the  $SO(10) \times$  flipped  $SU(5) \times U(1)'$  and  $SO(10) \times SU(5) \times U(1)$  models from the deconstruction of 5-dimensional supersymmetric orbifold  $SO(10)$  models, the discussions on how to break the gauge symmetry down to the diagonal flipped  $SU(5) \times U(1)'$  or  $SU(5) \times U(1)$  gauge symmetry are identical to those of Section III A. Two link fields with simple diagonal VEVs will leave the diagonal subgroup gauge bosons massless and give the other gauge bosons masses of order  $gv$ . We will consider the models inspired by deconstruction of the 5-dimensional  $SO(10)$  models with Wilson line gauge symmetry breaking, where the  $SO(10)$  gauge symmetry can be broken down to the flipped  $SU(5) \times U(1)'$  and  $SU(5) \times U(1)$ , and the 6-dimensional  $SO(10)$  model where the  $SO(10)$  gauge symmetry can be broken down to the SM like gauge symmetry.

Starting with the  $G \equiv SO(10) \times SO(10)$ , we use two bi-vectors  $U_1$  and  $U_2$  with quantum numbers  $(\mathbf{10}, \mathbf{10})$ .

In order to break the gauge symmetry down to the diagonal  $SU(5) \times U(1)$ , the VEVs of the link fields are assumed to be

$$\begin{aligned} \langle U_1 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_2 \rangle &= \frac{v}{\sqrt{2}} \sigma_2 \otimes \text{diag}(1, 1, 1, 1, 1) , \end{aligned}$$

and to obtain the flipped  $SU(5) \times U(1)'$ ,  $U_2$  VEV is replaced with

$$\langle U_2 \rangle = \frac{v}{\sqrt{2}} \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1) . \quad (48)$$

Note the similarity between the VEVs and the parity operators in Eqs. (14) and (15). As  $\langle U_1 \rangle$  commutes with  $T_a$  and  $T_{\hat{a}}$ , while  $\langle U_2 \rangle$  commutes with  $T_a$  and anti-commutes with  $T_{\hat{a}}$ , the gauge bosons of the unbroken subgroup  $G_s = SU(5) \times U(1)$  or flipped  $SU(5) \times U(1)'$  have a mass matrix identical to the right hand side of Eq. (44) while the  $G/G_s$  gauge bosons



have a mass matrix the same as the right hand side of Eq. (45). One set of the  $G_s$  gauge bosons remain massless and the  $G/G_s$  gauge bosons acquire masses of  $2gv$ .

Moreover, the  $SO(10) \times SO(10)$  can be broken to the intersection of two maximal subgroups,  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ . If we introduce four bi-vector link fields with VEVs

$$\begin{aligned} \langle U_1 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_2 \rangle &= \frac{v}{\sqrt{2}} \sigma_2 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_3 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_4 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1) , \end{aligned} \quad (49)$$

where all  $U_i$ 's are  $(\mathbf{10}, \mathbf{10})$  under the original group  $SO(10) \times SO(10)$ . It is easy to see that  $U_1$  and  $U_2$  are the same as the link fields we used for breaking  $SO(10) \times SO(10)$  to  $SU(5) \times U(1)$ , and  $U_3$  and  $U_4$  are the same as those for breaking  $SO(10) \times SO(10)$  to the PS group. To clarify the notation, we assign  $m_{SMX}$  for the mass matrices of the gauge bosons in the intersection of the  $SU(5) \times U(1)$  and PS gauge symmetry, which is  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ ,  $m_5$  for those in  $SU(5) \times U(1)$  but not in PS gauge group,  $m_{PS}$  for those in the PS gauge group but not in  $SU(5) \times U(1)$ , and  $m_R$  is reserved for the mass matrices of the rest gauge bosons belong to neither  $SU(5) \times U(1)$  or PS gauge group. These mass matrices are given by

$$m_{SMX}^2 = 4g^2v^2 \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} , \quad m_5^2 = m_{PS}^2 = 4g^2v^2 \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} , \quad m_R^2 = 4g^2v^2 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} . \quad (50)$$

It is obvious that only  $m_{SMX}$  has 0 eigenvalue, and the others do not. With the combination of these two sets of link fields, the  $SO(10) \times SO(10)$  gauge symmetry is broken to the  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$  gauge symmetry.

Similarly, we can break  $SO(10) \times SO(10)$  down to  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$  by choosing

$$\begin{aligned} \langle U_1 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_2 \rangle &= \frac{v}{\sqrt{2}} \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1) , \\ \langle U_3 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \end{aligned}$$

$$\langle U_4 \rangle = \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, -1, -1) , \quad (51)$$

or

$$\begin{aligned} \langle U_1 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_2 \rangle &= \frac{v}{\sqrt{2}} \sigma_2 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_3 \rangle &= \frac{v}{\sqrt{2}} \sigma_0 \otimes \text{diag}(1, 1, 1, 1, 1) , \\ \langle U_4 \rangle &= \frac{v}{\sqrt{2}} \sigma_2 \otimes \text{diag}(1, 1, 1, -1, -1) . \end{aligned} \quad (52)$$

In these models, the doublet-triplet splitting is still a problem without fine-tuning in the superpotential, thus it is difficult to solve the proton decay problem induced by the dimension-5 proton decay operators.

## V. $SO(10) \times SO(10)$ MODELS WITH BI-SPINOR LINK FIELDS

In this section, we demonstrate that, by using bi-spinor link fields  $(\mathbf{16}, \overline{\mathbf{16}})$  and  $(\overline{\mathbf{16}}, \mathbf{16})$ , we are able to break the gauge symmetry  $SO(10) \times SO(10)$  down to the diagonal  $SU(5) \times U(1)$  and flipped  $SU(5) \times U(1)'$  gauge symmetry, and at the same time solve the doublet-triplet splitting problem via the missing partner mechanism in the model with the flipped  $SU(5) \times U(1)'$  breaking chain. In this approach, we do not rely on the commutation relations between the link field VEVs and the generators.

We introduce two bi-fundamental fields  $U_1$  and  $U_2$  with following quantum numbers under  $SO(10) \times SO(10)$

|       | $SO(10)$                 | $SO(10)$                 |
|-------|--------------------------|--------------------------|
| $U_1$ | $\mathbf{16}$            | $\overline{\mathbf{16}}$ |
| $U_2$ | $\overline{\mathbf{16}}$ | $\mathbf{16}$            |

(53)

Then, the covariant derivative can be rewritten as, using  $\mathcal{A}_\mu$  defined in Appendix A,

$$\begin{aligned} D_\mu U_1 &= \partial_\mu U_1 - \frac{i}{\sqrt{2}} \mathcal{A}_\mu U_1 + \frac{i}{\sqrt{2}} \mathcal{B}_\mu U_1 , \\ D_\mu U_2 &= \partial_\mu U_2 + \frac{i}{\sqrt{2}} \mathcal{A}_\mu U_2 - \frac{i}{\sqrt{2}} \mathcal{B}_\mu U_2 . \end{aligned} \quad (54)$$

### A. $SU(5) \times U(1)$

The effective action for scalar components is the same as that in Eq. (24), and the bispinors  $U_1$  and  $U_2$  acquire the following VEVs

$$\langle U_1 \rangle = \langle U_2 \rangle = \frac{v}{\sqrt{2}} \text{diag}(0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1). \quad (55)$$

Note that the assignment of the 1's are consistent with the diagonal  $(\bar{\mathbf{5}}, \mathbf{5})$  and  $(\mathbf{1}, \mathbf{1})$  in the  $(\mathbf{16}, \bar{\mathbf{16}})$ , see Appendix A. The gauge fields in Eq. (A1) have the mass matrices of the following forms

$$m_\lambda^2 = m_V^2 = m_{W_L}^2 = m_Y^2 = g^2 v^2 X_1, \quad \text{and} \quad m_A^2 = m_{W_R}^2 = m_X^2 = 2g^2 v^2 X_2. \quad (56)$$

Thus, the massless fields are the 5 states in the  $\lambda$  fields, 6  $V$ , 2  $W_L$  and 12  $Y$  states, total of 25 independent states. They are the gauge bosons of the diagonal  $SU(5) \times U(1)$  group.

### B. Flipped $SU(5) \times U(1)'$

For the flipped  $SU(5) \times U(1)'$ , the  $\bar{\mathbf{5}}$  and  $\mathbf{1}$  consist of different fermion fields. The VEVs of  $U_1$  and  $U_2$  are chosen as

$$\langle U_1 \rangle = \langle U_2 \rangle = \frac{v}{\sqrt{2}} \text{diag}(0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0). \quad (57)$$

The mass matrices become

$$m_\lambda^2 = m_V^2 = m_{W_L}^2 = m_A^2 = g^2 v^2 X_1, \quad \text{and} \quad m_Y^2 = m_{W_R}^2 = m_X^2 = 2g^2 v^2 X_2, \quad (58)$$

and the massless states are 12  $A$ , 2  $W_L$ , 6  $V$  and the 5 states in  $\lambda$ 's. These 25 massless states are the gauge bosons of the diagonal flipped  $SU(5) \times U(1)'$ .

Three families of the SM fermions form three  $\mathbf{16}$  spinor representations under the first  $SO(10)$  while they are singlets under the second  $SO(10)$ . We also introduce a 10-dimensional Higgs field  $H_{\mathbf{10}}$ , and one pair of Higgs  $\Sigma_H$  and  $\bar{\Sigma}_H$  in the spinor representation. Moreover, we introduce one pair of the fields  $\chi$  and  $\bar{\chi}$  in the spinor representation, two singlet fields  $S$

and  $S'$ . The quantum numbers for these particles are

|                   | $SO(10)$            | $SO(10)$            |
|-------------------|---------------------|---------------------|
| $\mathbf{16}_i$   | $\mathbf{16}$       | $\mathbf{1}$        |
| $H_{\mathbf{10}}$ | $\mathbf{10}$       | $\mathbf{1}$        |
| $\Sigma_H$        | $\mathbf{16}$       | $\mathbf{1}$        |
| $\bar{\Sigma}_H$  | $\bar{\mathbf{16}}$ | $\mathbf{1}$        |
| $\chi$            | $\mathbf{1}$        | $\mathbf{16}$       |
| $\bar{\chi}$      | $\mathbf{1}$        | $\bar{\mathbf{16}}$ |
| $S, S'$           | $\mathbf{1}$        | $\mathbf{1}$        |

(59)

The superpotential is

$$\begin{aligned}
W = & y_1 \Sigma_H U_2 \bar{\chi} + y_2 \bar{\Sigma}_H U_1 \chi + y_3 S (\Sigma_H \bar{\Sigma}_H - M_{\text{FS}}^2) + y_4 S' H_{\mathbf{10}} H_{\mathbf{10}} + \lambda_x \Sigma_H H_{\mathbf{10}} \Sigma_H \\
& + \lambda_y \bar{\Sigma}_H H_{\mathbf{10}} \bar{\Sigma}_H + y_{ij} 16_i H_{\mathbf{10}} 16_j + \frac{y_5}{M_{\text{Pl}}} (U_1 U_2) (\bar{\chi} \chi) ,
\end{aligned}
\tag{60}$$

where  $M_{\text{FS}}$  is the unification scale for the flipped  $SU(5) \times U(1)'$ , and  $M_{\text{Pl}}$  is the Planck scale. The  $(\bar{\mathbf{5}} \oplus \mathbf{1})$  multiplets in  $\Sigma_H$  and  $\chi$ , and the  $(\mathbf{5} \oplus \mathbf{1})$  multiplets in  $\bar{\Sigma}_H$  and  $\bar{\chi}$  are assumed to have the  $SO(10)$  unification scale masses from the first two terms in the superpotential, while the  $\mathbf{10}$  multiplet  $\Sigma_h$  in  $\Sigma_H$ , the  $\mathbf{10}$  multiplet  $\chi_T$  in  $\chi$ , the  $\bar{\mathbf{10}}$  multiplet  $\bar{\Sigma}_h$  in  $\bar{\Sigma}_H$ , and the  $\bar{\mathbf{10}}$  multiplet  $\bar{\chi}_T$  in  $\bar{\chi}$  are still massless at  $M_*$ . For simplicity, we assume that the masses  $M_{\chi_T}$  for  $\chi_T$  and  $\bar{\chi}_T$  are about  $10^{12}$  GeV from the last term in superpotential. Under flipped  $SU(5) \times U(1)'$ ,  $H_{\mathbf{10}}$  is decomposed into one pair of 5-plets  $h$  and  $\bar{h}$ . To be explicit, we denote  $\Sigma_h$ ,  $\bar{\Sigma}_h$ ,  $h$  and  $\bar{h}$  as

$$\Sigma_h = (Q_H, D_H^c, N_H), \quad \bar{\Sigma}_h = (\bar{Q}_{\bar{H}}, \bar{D}_{\bar{H}}^c, \bar{N}_{\bar{H}}), \tag{61}$$

$$h = (D_h^r, D_h^g, D_h^b, H_d), \quad \bar{h} = (\bar{D}_h^r, \bar{D}_h^g, \bar{D}_h^b, H_u). \tag{62}$$

From the third, fifth and sixth terms in Eq. (60), we obtain the superpotential below  $M_*$

$$W = y_3 S (\bar{\Sigma}_h \Sigma_h - M_{\text{FS}}^2) + \lambda_x \Sigma_h \Sigma_h h + \lambda_y \bar{\Sigma}_h \bar{\Sigma}_h \bar{h} . \tag{63}$$

There is only one F-flat and D-flat direction, which can always be rotated along the  $N_H$  and  $\bar{N}_{\bar{H}}$  directions. Hence we obtain that  $\langle N_H \rangle = \langle \bar{N}_{\bar{H}} \rangle = M_{\text{FS}}$ . In addition, the superfields in  $\Sigma_h$  and  $\bar{\Sigma}_h$  are eaten or acquire large masses via the supersymmetric Higgs mechanism,

except for  $D_H^c$  and  $\bar{D}_H^c$ . The superpotential  $\lambda_x \Sigma_h \Sigma_h h$  and  $\lambda_y \bar{\Sigma}_h \bar{\Sigma}_h \bar{h}$  couple the  $D_H^c$  and  $\bar{D}_H^c$  with the  $D_h$  and  $\bar{D}_h$  respectively to form the massive eigenstates with masses  $\lambda_x < N_H >$  and  $\lambda_y < \bar{N}_H >$ . We naturally have the doublet-triplet splitting due to the missing partner mechanism [29, 30, 31]. Because the triplets in  $h$  and  $\bar{h}$  only have small mixing through the  $\mu$  term, the higgsino-exchange mediated proton decay are negligible, *i.e.*, we do not have the dimension-5 proton decay problem. Proton decay via the dimension-6 operators is well above the current experimental bounds.

The gauge coupling unification for the flipped  $SU(5) \times U(1)'$  is realized by first unifying  $\alpha_2$  and  $\alpha_3$  at scale  $M_{23}$ , then the gauge couplings of  $SU(5)$  and  $U(1)'$  further unify at  $M_*$ . From  $M_Z$  to  $M_{SUSY}$ , the beta functions are  $b^0 \equiv (b_1, b_2, b_3) = (41/10, -19/6, -7)$ , and from  $M_{SUSY}$  to  $M_{\chi_T} = 10^{12}$  GeV, the beta functions are  $b^I = (33/5, 1, -3)$ . From  $M_{\chi_T}$  to the  $\alpha_2$  and  $\alpha_3$  unification scale  $M_{23}$ , the beta functions are  $b^{II} = (36/5, 4, 0)$ , because  $\chi_T$  and  $\bar{\chi}_T$  contribute to the gauge coupling RGE runnings.

Unification of  $\alpha_2$  and  $\alpha_3$  at  $M_{23}$  gives the condition

$$\begin{aligned} \alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z) = & \frac{b_2^0 - b_3^0}{2\pi} \log \left( \frac{M_{SUSY}}{M_Z} \right) + \frac{b_2^I - b_3^I}{2\pi} \log \left( \frac{M_{\chi_T}}{M_{SUSY}} \right) \\ & + \frac{b_2^{II} - b_3^{II}}{2\pi} \log \left( \frac{M_{23}}{M_{\chi_T}} \right), \end{aligned} \quad (64)$$

which can be solved for  $M_{23}$ .

The coupling  $\alpha'_1$  of  $U(1)'$  is related to  $\alpha_1$  and  $\alpha_5$  at  $M_{23}$  by

$$\alpha'^{-1}_1(M_{23}) = \frac{25}{24} \alpha_1^{-1}(M_{23}) - \frac{1}{24} \alpha_5^{-1}(M_{23}). \quad (65)$$

Between  $M_{23}$  and  $M_*$ , besides the 3 families of the Standard Model fermions  $\mathbf{16}_i$ , there are the  $SU(5) \times U(1)'$  gauge bosons in  $\mathbf{24} + \mathbf{1}$ ,  $H_{\mathbf{10}}$ ,  $\Sigma_h$ ,  $\bar{\Sigma}_h$ ,  $\chi_T$  and  $\bar{\chi}_T$ , thus the beta functions for  $U(1)'$  and  $SU(5)$  are  $b^{III} \equiv (b'_1, b_5) = (8, -2)$ .

In Fig. 2, we show the runnings of the gauge couplings near the unification scale  $M_*$ . The unification scale  $M_{23}$  for  $SU(2)_L$  and  $SU(3)_C$  is about  $2.6 \times 10^{16}$  GeV, and the  $SO(10) \times SO(10)$  unification scale  $M_*$  is about  $2.0 \times 10^{17}$  GeV. If the masses  $M_{\chi_T}$  for  $\chi_T$  and  $\bar{\chi}_T$  are larger than  $M_{23}$ , we will have  $\alpha'_1 > \alpha_5$  at  $M_{23}$ . Because above  $M_{23}$  the beta function of  $U(1)'$  is positive while that of  $SU(5)$  is negative, we can not achieve the unification of these two gauge couplings. In short, the intermediate mass scale  $M_{\chi_T}$  is important for the gauge coupling unification [32].

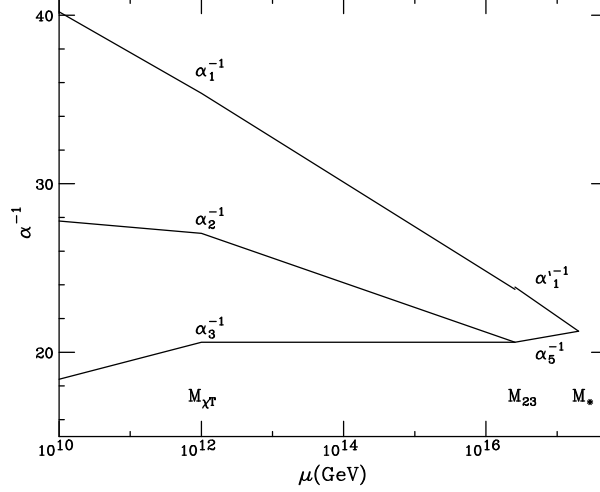


FIG. 2: The gauge coupling unification near  $M_* = 2.0 \times 10^{17}$  GeV for flipped  $SU(5) \times U(1)'$  model.

### C. Brief Comments on $SO(10) \times$ Flipped $SU(5) \times U(1)'$

We introduce the link fields  $U_1$ ,  $U_2$ ,  $U_3$  and  $U_4$  with following quantum numbers

|       | $SO(10)$                 | $SU(5) \times U(1)'$              |      |
|-------|--------------------------|-----------------------------------|------|
| $U_1$ | <b>16</b>                | <b>(5, 3)</b>                     | (66) |
| $U_2$ | $\overline{\mathbf{16}}$ | <b>(<math>\bar{5}</math>, -3)</b> |      |
| $U_3$ | <b>16</b>                | <b>(1, -5)</b>                    |      |
| $U_4$ | $\overline{\mathbf{16}}$ | <b>(1, 5)</b>                     |      |

We choose the following VEVs for the link fields  $\langle U_1 \rangle = \langle U_2 \rangle = K_{16 \times 5}$  and  $\langle U_3 \rangle = \langle U_4 \rangle = L_{16 \times 1}$ , where  $K_{16 \times 5}$  is a  $16 \times 5$  matrix with elements at (4, 4), (8, 5), (13, 1), (14, 2), and (15, 3) being  $v/\sqrt{2}$  and all other elements 0, and  $L_{16 \times 1}$  is a column vector with  $v/\sqrt{2}$  as the (12, 1) element and all other elements 0. Similarly to above subsection, we can break the  $SO(10) \times$  flipped  $SU(5) \times U(1)'$  gauge symmetry down to a diagonal flipped  $SU(5) \times U(1)'$  gauge symmetry.

Three families of the SM fermions form three **16** spinor representations under the first  $SO(10)$  while they are singlets and neutral under the flipped  $SU(5) \times U(1)'$ . We also introduce a 10-dimensional Higgs field  $H_{10}$ , and one pair of Higgs  $\Sigma_H$  and  $\bar{\Sigma}_H$  in the spinor representation. Moreover, we introduce one pair of the 5-plet fields  $\chi_5$  and  $\bar{\chi}_5$ , one pair of the 10-plet fields  $\chi_T$  and  $\bar{\chi}_T$ , one pair of the  $U(1)'$  charged singlets  $S_c$  and  $\bar{S}_c$ , and two

singlet fields  $S$  and  $S'$ . The quantum numbers for these particles are

|                   | $SO(10)$            | $SU(5) \times U(1)'$               |
|-------------------|---------------------|------------------------------------|
| $\mathbf{16}_i$   | $\mathbf{16}$       | $(\mathbf{1}, \mathbf{0})$         |
| $H_{\mathbf{10}}$ | $\mathbf{10}$       | $(\mathbf{1}, \mathbf{0})$         |
| $\Sigma_H$        | $\mathbf{16}$       | $(\mathbf{1}, \mathbf{0})$         |
| $\bar{\Sigma}_H$  | $\bar{\mathbf{16}}$ | $(\mathbf{1}, \mathbf{0})$         |
| $\chi_5$          | $\mathbf{1}$        | $(\bar{\mathbf{5}}, -\mathbf{3})$  |
| $\bar{\chi}_5$    | $\mathbf{1}$        | $(\mathbf{5}, \mathbf{3})$         |
| $\chi_T$          | $\mathbf{1}$        | $(\mathbf{10}, \mathbf{1})$        |
| $\bar{\chi}_T$    | $\mathbf{1}$        | $(\bar{\mathbf{10}}, -\mathbf{1})$ |
| $S_s$             | $\mathbf{1}$        | $(\mathbf{1}, \mathbf{5})$         |
| $\bar{S}_s$       | $\mathbf{1}$        | $(\mathbf{1}, -\mathbf{5})$        |
| $S, S'$           | $\mathbf{1}$        | $(\mathbf{1}, \mathbf{0})$         |

(67)

The superpotential is

$$\begin{aligned}
W = & y_1 \Sigma_H U_2 \bar{\chi}_5 + y_2 \bar{\Sigma}_H U_1 \chi_5 + y_3 \Sigma_H U_4 \bar{S}_s + y_4 \bar{\Sigma}_H U_3 S_s + y_5 S (\Sigma_H \bar{\Sigma}_H - M_{\text{FS}}^2) + \lambda_x \Sigma_H H_{\mathbf{10}} \Sigma_H \\
& + \lambda_y \bar{\Sigma}_H H_{\mathbf{10}} \bar{\Sigma}_H + y_{ij} 16_i H_{\mathbf{10}} 16_j + y_6 S' H_{\mathbf{10}} H_{\mathbf{10}} + \frac{1}{M_{\text{Pl}}} (y_7 U_1 U_2 + y_8 U_3 U_4) \chi_T \bar{\chi}_T .
\end{aligned} \tag{68}$$

The discussions for the proton decay and the gauge coupling unification are similar to these in the previous subsection. We want to emphasize that the doublet-triplet splitting problem can be solved by the missing partner mechanism, and the solution of the proton decay problem follows.

## VI. CONCLUSIONS

We have considered the 4-dimensional  $N = 1$  supersymmetric  $SO(10)$  models inspired by the deconstruction of 5-dimensional supersymmetric orbifold  $SO(10)$  models and high dimensional non-supersymmetric  $SO(10)$  models with Wilson line gauge symmetry breaking. We studied the  $SO(10) \times SO(10)$  models with bi-fundamental link fields where the  $SO(10) \times SO(10)$  gauge symmetry can be broken down to the PS,  $SU(5) \times U(1)$ , flipped  $SU(5) \times U(1)'$  and the SM like gauge symmetry. However, we need to fine-tune the superpotential to obtain the doublet-triplet splitting. We proposed two interesting models: the  $SO(10) \times SO(6) \times$

$SO(4)$  model where the gauge symmetry can be broken down to PS gauge symmetry, and the  $SO(10) \times SO(10)$  model with bi-spinor link fields in which the gauge symmetry can be broken down to the flipped  $SU(5) \times U(1)'$  gauge symmetry. These intermediate gauge symmetries can be further broken down to the SM gauge symmetry. In these models, the missing partner mechanism can be implemented to solve the doublet-triplet splitting problem, then the higgsino-exchange mediated proton decay is negligible, and consequently the proton decay is mainly induced by the dimension-6 operators. With the GUT scale being at least a few times  $10^{16}$  GeV, the proton lifetime is well above the current experimental bound. In addition, we discussed the gauge coupling unification in these two models: in the  $SO(10) \times SO(6) \times SO(4)$  models, the gauge couplings unify at  $3.3 \times 10^{16}$  GeV for a left-right symmetric model and at  $5.1 \times 10^{16}$  GeV for a left-right non-symmetric model; and in the  $SO(10) \times SO(10)$  model, the gauge couplings unify at about  $2.0 \times 10^{17}$  GeV. Furthermore, we briefly commented on the interesting variation models with gauge groups  $SO(10) \times SO(6)$  and  $SO(10) \times$  flipped  $SU(5) \times U(1)'$  where the proton decay problem can be solved in the similar manner, and the  $SO(10) \times SO(10)$  model with bi-spinor link fields in which the gauge symmetry can be broken down to the  $SU(5) \times U(1)$  gauge symmetry.

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### APPENDIX A

The generators and the assignment of the fermions in the **16** can be found in Ref. [33]. We copy the  $\sigma \cdot W_\mu$ , and rename it  $\mathcal{A}_\mu$ . The  $16 \times 16$  matrix can be re-written into four  $8 \times 8$  matrices,

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{pmatrix}, \quad (\text{A1})$$



with

$$\begin{aligned}
\mathcal{A}_{11} &= \begin{pmatrix} \lambda_{11} & V_{12} & V_{13} & X_1^0 & W_L^- & & & \\ V_{12}^* & \lambda_{22} & V_{23} & X_2^- & & W_L^- & & \\ V_{13}^* & V_{23}^* & \lambda_{33} & X_3^- & & & W_L^- & \\ \overline{X}_1^0 & X_2^+ & X_3^+ & \lambda_{44} & & & & W_L^- \\ W_L^+ & & & & \lambda_{55} & V_{12} & V_{13} & X_1^0 \\ & W_L^+ & & & V_{12}^* & \lambda_{66} & V_{23} & X_2^- \\ & & W_L^+ & & V_{13}^* & V_{23}^* & \lambda_{77} & X_3^- \\ & & & W_L^+ & \overline{X}_1^0 & X_2^+ & X_3^+ & \lambda_{88} \end{pmatrix} \\
\mathcal{A}_{12} &= \begin{pmatrix} 0 & A_6^0 & -A_5^0 & -Y_1^+ & 0 & -Y_6^- & Y_5^- & -\overline{A}_1^0 \\ -A_6^0 & 0 & A_4^- & -\overline{Y}_2^0 & Y_6^- & 0 & -Y_4^{--} & -A_2^- \\ A_5^0 & -A_4^- & 0 & -\overline{Y}_3^0 & -Y_5^- & Y_4^{--} & 0 & -A_3^- \\ Y_1^+ & \overline{Y}_2^0 & \overline{Y}_3^0 & 0 & \overline{A}_1^0 & A_2^- & A_3^- & 0 \\ 0 & -A_3^+ & A_2^+ & -Y_4^{++} & 0 & Y_3^0 & -Y_2^0 & -A_4^+ \\ A_3^+ & 0 & A_1^0 & -Y_5^+ & Y_3^0 & 0 & -Y_1^- & -\overline{A}_5^0 \\ -A_2^+ & A_1^0 & 0 & -Y_6^+ & Y_2^0 & -Y_1^- & 0 & -\overline{A}_6^0 \\ Y_4^{++} & Y_5^+ & Y_6^+ & 0 & A_4^+ & \overline{A}_5^0 & \overline{A}_6^0 & 0 \end{pmatrix} \\
\mathcal{A}_{21} &= \begin{pmatrix} 0 & -\overline{A}_6^0 & \overline{A}_5^0 & Y_1^- & 0 & A_3^- & -A_2^- & Y_4^{--} \\ \overline{A}_6^0 & 0 & -A_4^+ & Y_2^0 & -A_3^- & 0 & \overline{A}_1^0 & Y_5^- \\ -\overline{A}_5^0 & A_4^+ & 0 & Y_3^0 & A_2^- & -\overline{A}_1^0 & 0 & Y_6^- \\ -Y_1^- & -Y_2^0 & -Y_3^0 & 0 & -Y_4^- & -Y_5^- & -Y_6^- & 0 \\ 0 & Y_6^+ & -Y_5^+ & A_1^0 & 0 & -\overline{Y}_3^0 & \overline{Y}_2^0 & A_4^- \\ -Y_6^+ & 0 & Y_4^{++} & A_2^+ & \overline{Y}_3^0 & 0 & -Y_1^+ & A_5^0 \\ Y_5^+ & -Y_4^{++} & 0 & A_3^+ & -\overline{Y}_2^0 & Y_1^+ & 0 & A_6^0 \\ -A_1^0 & -A_2^+ & -A_3^+ & 0 & -A_4^- & -A_5^0 & -A_6^0 & 0 \end{pmatrix}
\end{aligned}$$

$$A_{22} = \begin{pmatrix} \lambda_{99} & -V_{12}^* & -V_{13}^* & -\overline{X}_1^0 & W_R^- & & & & \\ -V_{12} & \lambda_{1010} & -V_{23}^* & -X_2^+ & & W_R^- & & & \\ -V_{13} & -V_{23} & \lambda_{1111} & -X_3^+ & & & W_R^- & & \\ -X_1^0 & -X_2^- & -X_3^- & \lambda_{1212} & & & & W_R^- & \\ W_R^+ & & & & \lambda_{1313} & -V_{12}^* & -V_{13}^* & -\overline{X}_1^0 & \\ & W_R^+ & & & -V_{12} & \lambda_{1414} & -V_{23}^* & -X_2^+ & \\ & & W_R^+ & & -V_{13} & -V_{23} & \lambda_{1515} & -X_3^+ & \\ & & & W_R^+ & -X_1^0 & -X_2^- & -X_3^3 & \lambda_{1616} & \end{pmatrix}$$

The 45 gauge bosons consist of 12  $A$ , 6  $X$ , 6  $V$ , 12  $Y$ , 2 charge  $W_L$ , 2 charge  $W_R$  and 16  $\lambda$  which can be rewritten as 5 independent fields,  $V_3$ ,  $V_8$ ,  $V_{15}$ ,  $W_L^0$  and  $W_R^0$ .

The first family of the SM fermions forms a **16**,

$$\mathbf{16}_1 = (u_r, u_g, u_b, \nu_e, d_r, d_g, d_b, e^-, d_r^c, d_g^c, d_b^c, e^+, -u_r^c, -u_g^c, -u_b^c, -\nu_e^c)^T, \quad (\text{A2})$$

similarly for the second and third families. As the  $SO(10)$  is broken down to  $SU(5) \times U(1)$  or flipped  $SU(5) \times U(1)'$ , the spinor representation **16** is decomposed as

$$\mathbf{16} \rightarrow \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1} \quad (\text{A3})$$

where

$$\mathbf{10} = (Q, U^c, e^+), \quad \bar{\mathbf{5}} = (D^c, L), \quad \text{and} \quad \mathbf{1} = \nu^c \quad (\text{A4})$$

for breaking to  $SU(5) \times U(1)$ , and

$$\mathbf{10} = (Q, D^c, \nu^c), \quad \bar{\mathbf{5}} = (U^c, L), \quad \text{and} \quad \mathbf{1} = e^+ \quad (\text{A5})$$

for breaking to flipped  $SU(5) \times U(1)'$ .

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